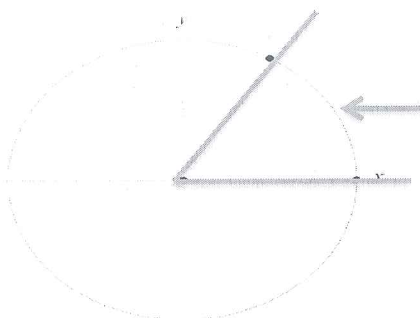
Sec. 8.1 Radians and Arc Length

Central Angle – angle whose vertex is the center of a circle

Radian - One radian is the angle at the center of a circle subtended by an arc that is equal in length to the radius of the circle.

In a unit circle, the radius would be 1, and the angle created (subtended) by an arc length of 1, would be 1 radian. A radian is the angle created by the arc length, not the arc length!



The measure of any angle in radians will be equal to the ratio of the arc length subtended by that angle (s) and the length of the radius (r): $\theta = \frac{s}{r}$.

Radians!

Arc Length – For a circle with radius r, a central angle of θ radians subtends an arc whose length s is given by $s = r\theta$.

θ = must be in radians

Ex: Find the measure (in radians) of θ if it is creates an arc length of 12 cm and has a radius of 5cm.

$\theta = \frac{s}{r}$

$\theta = \frac{12}{5}$

$\theta = 2.4$ radians

Ex: Find the length of the arc of a circle of radius 2 meters subtended by a central angle of .25 radians.

$s = r\theta$

$s = 2(.25)$

$s = .5$ units

NOTE: a. 1 revolution = 2π radians
b. 180 degrees = π radians

Converting degrees to radians and radians to degrees use the fact that $180^\circ = \pi$ to set up and solve a proportion.

$$\frac{\theta^\circ}{180} = \frac{\theta\,(\text{rad})}{\pi}$$

Ex: Convert each of the following:

a. 60 degrees

$$\frac{60^\circ}{180^\circ} = \frac{\theta}{\pi}$$
$$\frac{180\theta}{180} = \frac{60\pi}{180}$$
Exact → $\boxed{\theta = \frac{\pi}{3} \text{ radians}}$
Approx → $\boxed{\theta = 1.047 \text{ radians}}$

b. –45 degrees

$$\frac{-45^\circ}{180^\circ} = \frac{\theta}{\pi}$$
$$-45\pi = 180\theta$$
$$\frac{-45\pi}{180} = \theta$$
$\boxed{-\frac{\pi}{4} \text{ rad} = \theta \quad .785 \text{ rad} = \theta}$

c. $\pi/6$ radians

$$\frac{\theta}{180} = \frac{\frac{\pi}{6}}{\pi}$$
$$\theta\pi = 180 \cdot \frac{\pi}{6}$$
$$\theta\pi = 30\pi$$
$\boxed{\theta = 30^\circ}$

d. $-3\pi/4$ radians

$$\frac{\theta}{180} = \frac{-\frac{3\pi}{4}}{\pi}$$
$$\pi\theta = -\frac{3}{4}\pi \cdot 180$$
$$\pi\theta = -3(45)\pi$$
$\boxed{\theta = -135^\circ}$

Degrees	0	30	45	60	90	120	135	150	180
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
Degrees		210	225	240	270	300	315	330	360
Radians		$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π

Ex: a. Convert 3 radians to degrees.

$$\frac{\theta}{180} = \frac{3}{\pi}$$
$$\frac{\pi\theta}{\pi} = \frac{540}{\pi}$$
$\boxed{\theta = 171.887^\circ}$

b. Convert 3 degrees to radians

$$\frac{3}{180} = \frac{\theta}{\pi}$$
$$\frac{180\theta}{180} = \frac{3\pi}{180}$$
$\boxed{\theta = \frac{\pi}{60} \text{ radians} \quad \theta = .052 \text{ rad}}$

Ex: You walk 4 miles around a circular lake. Give an angle in radians that represents your final position relative to your starting point if the radius of the lake is:

(a) 1 mile

$$\theta = \frac{s}{r}$$
$$\theta = \frac{4}{1}$$
$\boxed{\theta = 4 \text{ radians}}$

(b) 3 miles

$$\theta = \frac{s}{r}$$
$$\theta = \frac{4}{3}$$
$\boxed{\theta = 1\tfrac{1}{3} \text{ radians}}$

Ex: Evaluate: (a) $\cos 3.14^\circ$

.9985

CLOSE TO 0°

$\cos 0^\circ = 1$

(b) cos 3.14

–.99999873

CLOSE TO π RADIANS OR 180°

$\cos 180^\circ = -1$

Area of a Sector – the area A of a sector of a circle of radius r formed by a central angle of θ radians is $A = \frac{1}{2}r^2\theta$. Must be in radians (θ).

exact

Ex: Find the area of a sector of a circle of radius 2 feet formed by an angle of 30 degrees.

② $A = \frac{1}{2}(2)^2\left(\frac{\pi}{6}\right)$
$= \frac{1}{2}(4)\left(\frac{\pi}{6}\right)$
$= 2 \cdot \frac{\pi}{6}$
$\boxed{A = \frac{\pi}{3}\ ft^2}$

① $\frac{30^\circ}{180^\circ} = \frac{\theta}{\pi}$
$\frac{180\theta}{180} = \frac{30\pi}{180}$
$\theta = \frac{\pi}{6}$

Circular motion:

Linear velocity – if *s* is the distance traveled in time *t* around this circle, then the speed, *v*, is defined as $v = \frac{s}{t}$.

Angular velocity – defined by w is the angle measured in radians divided by the elapsed time, i.e. $w = \frac{\theta}{t}$.

Converting Between Linear and Angular Velocities: $v = rw$ where *w* is measured in radians per unit time.

Ex: A child is spinning a rock at the end of a 2 foot rope at the rate of 150 revolutions per minute. What is its angular velocity? Find the linear velocity of the rock when it is released.

Angular velocity: $\omega = \frac{\theta\,(rad)}{t}$
$= \frac{150 \cdot 2\pi}{min}$
$\boxed{\omega = \frac{300\pi}{min}}$

$V = r\omega$
$= 2(300\pi)$
$\boxed{V = 600\pi\ ft/min}$

HW: pg 322 -324, #3-48 m/3